647510 **BSc4MatC401x** **Seat No :________**

B.Sc. Semester - 4 (*CBCS*) Examination
March/April-2024 [NEW COURSE]
Mathematics-M401(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any two of the following questions. (10)

(1) Show that the sequence $\{x_n\}$ defined by $x_1 = \sqrt{3}$ and $x_{n+1} = \sqrt{3x_n}$ $(n \geq 1)$ is convergent and also find its limit.

(2) Prove that point wise product of two convergent sequences is convergent.

(3) Define limit point of a sequence and find all limit points of the sequence $\{x_n\}$, where $x_n = (-1)^n, n \in \mathbb{N}$.

Que.1(B) Answer any one of the following questions. (04)

(1) Let $\{ x_n \}$ be a sequence of non-negative real numbers converging to x. Prove that $x \geq 0$.

(2) Evaluate: $\lim\limits_{n\to\infty} \frac{1}{2n}\left(1 + 2^{\frac{1}{2}} + 3^{\frac{1}{3}} + . . + n^{\frac{1}{n}}\right)$.

Que.2(A) Answer any two of the following questions. (10)

(1) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$.

(2) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{(n!)^2\, x^n}{(2n)!}$.

(3) Prove or disprove: Convergent series is always absolutely convergent.

Que.2(B) Answer any one of the following questions. (04)

(1) State comparison test, limit form of comparison test and Leibnitz test.

(2) Show that the series $\frac{1}{2} - \frac{2}{5} + \frac{3}{10} - \frac{4}{17} + . . .$ is conditionally convergent.

Que.3(A) Answer any two of the following questions. (10)

(1) Let $B = \{u_1, u_2, \ \ldots \ , u_n\}$ be a basis of a vector space U and $v_1, v_2, \ \ldots \ , v_n$ be vectors of a vector space V. Then prove that there exists a unique linear transformation $T : U \to V$such that $T(u_i) = v_i \ \ \forall\, i = 1, 2, \ldots, n$.

(2) Verify rank nullity theorem for $T : \mathbb{R}^4 \to \mathbb{R}^3$ defined as

$$T(x_1, x_2, x_3, x_4) = (x_1 + x_2 + x_3 + x_4, x_1 - x_2, x_1 + x_2 + x_4),\ \forall\, (x_1, x_2, x_3, x_4) \in \mathbb{R}^4.$$

(3) Define kernel of a linear transformation and show that it is a subspace of the domain of the linear transformation.

Que.3(B) Answer any one of the following questions. (04)

(1) State and prove necessary and sufficient condition for a linear transformation $T : U \to V$ to be one-one.

(2) Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be defined as

$T(x_1, x_2, x_3) = (x_1 + x_2, x_2, -x_3, 2x_1 + 5x_2 - 3x_3), \forall (x_1, x_2, x_3) \in \mathbb{R}^3$.

Let B_1, B_2 be the standard basis of $\mathbb{R}^3$ and $\mathbb{R}^4$, respectively. Find $[T : B_1, B_2]$.

Que.4(A) Answer any two of the following questions. (10)

(1) State and prove Cauchy – Schwartz inequality.

(2) Let $B = \{(1,0,-1), (0,0,1), (0,1,0)\}$ be a basis of $\mathbb{R}^3$. Find an orthonormal basis of $\mathbb{R}^3$ using Gram Schmidt orthogonalization process.

(3) Let $u, v \in V$ such that $||u|| = 3, ||u + v|| = 4, ||u - v|| = 6$. Find $||v||$.

Que.4(B) Answer any one of the following questions. (04)

(1) Define inner product space and orthogonal set.

(2) Prove that two vectors u and v in an inner product space V are orthogonal if and only if $|| u + v ||^2 = ||u||^2 + ||v||^2$.

Que.5(A) Answer any two of the following questions. (10)

(1) Derive formula to find radius of curvature when equation of a curve is given in polar form.

(2) Show that points of inflexion of the curve $y^2 = (x - a)^2(x - b)$ lie on the line $3x + a = 4b$.

(3) Find all asymptotes to the curve $4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 = 1$.

Que.5(B) Answer any one of the following questions. (04)

(1) Find radius of curvature at origin of the curve $x^2 + 2xy + 2y^2 - 4x = 0$.

(2) Find position and nature of the double point of the curve $x^3 + x^2 + y^2 - x - 4y + 3 = 0$.

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